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MAMDANI'S KNOWLEDGE BASE AND FUZZY RULES APPROXIMATION WITHIN FLb

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Abstract. We consider the problem of obtaining of fuzzy rule of Mamdani's approximation as the deduction of special axioms. Fuzzy logic in a broad sense (FLb) is used to solve the problem. A method for recognition, by which a special class of linguistic syntagms is determined in FLb, based on natural language. Also represented a method to model combinations of informative signs, through which pattern recognition takes place.

Keywords: fuzzy logic in a broad sense, syntagm, predicate, intensity, linguistic description.

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1. Introduction

Nowadays the formalization of intellectual operations, modeling human fuzzy statements about states and behavior of complex phenomena, forms an independent branch of science and applied researches, called "fuzzy modeling". The main problems, which can be solved in fuzzy modeling, related to simulation of intellectual operations of human approximate reasoning. According to famous theorem FAT (Fuzzy Approximation Theorem) proved by Bartolomeo Cosco, any mathematical system can be approximated by a system based on fuzzy logic. In other words, by natural language sentences "if-then" with their subsequent formalization by resources of fuzzy set theory we can accurately describe any number of arbitrary relationships "inputs-output" without using of complicated apparatus of differential and integral calculus, traditionally used in the management and identification. Among the intellectual operations that are used in human approximate reasoning there are the definition of semantics, the meaning of utterance, the definition of "verity" of elementary and compound utterance, the deduction of "verity" of statements on the basis of logical reasoning. The formalization of fuzzy implication allowed to set "if-then" rules in the form of fuzzy production rules and laid the foundation for fuzzy modeling experience and expert knowledge, expressed in terms of approximate dependences [1]. The concept of fuzzy logic is typically used in two senses - narrow and broad. In a narrow sense, fuzzy logic is a logic system that is an extension of multivalued logic. In a broad sense, which now prevails, fuzzy logic is equivalent to the theory of fuzzy sets, ie, classes with the inaccurate blurred boundaries [2]. Fuzzy logic in a broad sense (FLb) has the ability to extend classical logic in areas where classical logic can not provide satisfactory solutions. There are some problems associated with natural language, by which a better model can be build in FLb than it is possible in classical logic, because FLb aims to create a mathematical model of natural human reasoning, in which the human language played fundamental role.

2. Statement of the Problem

When building a system of pattern recognition is necessary to solve three major problems:

- selection of informative features;
- modeling of object
- making a decision - issuing some control action, for example, on the basis of hard-coded rules, "If (condition) then (action)."

The paper presents the new approach to all three tasks in terms of FLb, which arise in the construction of systems of pattern recognition.

Consider the scheme of fuzzy rule-based Mamdani's deduction [3,4]. Let the input variable x is any fuzzy set A, and y - fuzzy set B, then the output variable is a fuzzy set. If the fact $x = x_0$, $y = y_0$ is specified then it is necessary to find $z = z_0$.

3. Methods of solution

In order to construct algorithms and programs in pattern recognition problems, it should be set rules for dealing with units of different levels of natural language [5], because the definition of informative features in pattern recognition tasks is one of the most important and responsible steps. The main problem is to find a set of features that best reflects the concept of similarity.

Variation of the <u>informativity</u> of a <u>feature</u> performed by various ways, main among them is the *syntagmatic segmentation*. Generalized linguistic units are defined by the notion of syntagm, and the syntagmatic descriptions may reflect one or another specificity of research. The most informative elements of meaning describing the relationship are only at the level of syntagms, the allocation of which requires the use of non-trivial algorithm of syntactic analysis [6].

The syntagm may consist of one word, may coincide with whole sentence. It also may or may not coincide with the combination of words, but there significant differences remain between them: syntagm stands out in the sentence <u>and</u> is the result of its division and exists only in it, while the combination of words is not only stands out in the sentence, but in addition with the word serves as a ready-made "building material" for sentence and is not the result of decomposition to the elements, but is synthesis of elements [7].

Syntagms have great informative congestion: they contain an additional message that accompanies the report, contained in the distributed parts of the sentence, and are characterized by relative informative independence.

There are three concepts in a logical analysis of natural language: intensity, extension and possible world.

Possible world is called the category of modal logic used to determine the truth or falsity of modal statements. In general terms, a possible world can be interpreted as a possible state of causes or possible developments of events. Possible world can be expanded by involving those linguistic resources which provide the right of choice when interpreting the meaning of spoken [8].

Intensity is the set of conceivable features of the signified by concept object or phenomenon, which can lead to different values of truth in different possible worlds [9]. In logic, the intensity is a function associated with the value of truth to the object in every possible world.

Extension is the set of elements defined by a single intensity, which is a value of this syntagm in given possible world [9].

Definition 1. Let A is an estimated syntagm from some set S of syntagms, predicates and clauses. Then syntagm

$$\langle Noun \rangle \text{ is } \mathrm{A}$$

is ***estimated predicate***.

We fix a many-sorted language J, which has a finite number of varieties l and associate the syntagm from S to the elements of J. F_J is the set of correctly formed formulas of the corresponding syntagms.

Definition 2. Let A $\in S$ is a syntagm, and $\mathrm{A}(x_1,...,x_n) \in F_j$ is the corresponding formula. Then the set

$$\mathrm{A}_{(x_1,...,x_n)} = \{a_{t_1,...,t_n} / \mathrm{A}_{x_1,...,x_n}[t_1,...,t_n] \mid t_1 \in M_{l_1},...,t_n \in M_{l_n}\},$$

called ***multiformula*** is the intensity of A , M_{l_1}, M_{l_2} - the set of terms of computable formulas.

Definition 3. Let A $_i \in S$, $i=1,...,m$ are syntagms with the intensities A_i . ***Formal theory FLb*** is

$$\mathrm{T} = \{ \mathrm{A}_0[\mathrm{A}_0],…, \mathrm{A}_m[\mathrm{A}_m]\}. \quad (1)$$

Since the intensities are multiformulas, the theory T in (1) adjoins to the fuzzy theory in narrow sense (FLn) T:

$$T = ST \cup \mathrm{A}_0 \cup … \cup \mathrm{A}_m$$

where ST is the supported fuzzy theory [10]. Thus, all basic operations of FLb can be transformed into FLn.

We form the general scheme of formal theory, using natural language, at the same time it is associated with a fuzzy theory T in FLn. Then, deductions are produced in FLb. The multiformula, which can be regarded as the most accurate intensity of corresponding syntagm and exists as deduction in FLn will be the result.

We introduce a special syntagm

R := "<*noun*>$_1$ *in relation with* "<*noun*>$_2$"

with intensity

$$\mathrm{R}_{\langle x,y\rangle}=\{r_{ts}/\mathrm{R}_{x,y}[t,s] \mid t\in M_{l_1}, s\in M_{l_2}\},$$

where R is some binary predicate symbol.

Let T is a theory in FLb. Linguistic assertion A, which can be conditional clause is true in T, if it has an intensity

$$\mathrm{A}_{(x_1,\dots,x_n)}=\{1/\,\mathrm{A}_{x_1,\dots,x_n}[t_1,\dots,t_n] \mid t_1\in M_{l_1},\dots,t_n\in M_{l_n}\}.$$

Let we have two estimated predicates, between them there is some relation R

"<*noun*>$_1$ *is* A *and* <*noun*>$_2$ *is* B " ,

where <*noun*>$_1$ is the name of the first element of each pair, and <*noun*>$_2$ - the name of the second. These predicates can be described in natural language with using of conditional clauses

P :=IF <*noun*>$_1$ is A *and* <*noun*>$_2$ *is* B , THEN R . (4)

A special role in fuzzy logic belongs to conditional clauses

IF A , THEN B.

This clauses are implications described in natural language and used to describe the dynamic processes or models of decision making. The set of such assertions is called *linguistic description*.

Let A_j, B_j are estimated predicates with the corresponding intensities A_j, B_j. Then the linguistic description in FLb is the finite set LD^I or the finite set LD^A of following statements:

$$\mathrm{LD}^I=\{\mathrm{R}_1^I,\dots,\mathrm{R}_m^I\},$$

where R_j^I =IF A_j *THEN* B_j, $j=1,\dots,m$ are conditional clause,

and

$$\mathrm{LD}^A=\{\mathrm{R}_1^A,\dots,\mathrm{R}_m^A\},$$

where $\mathrm{R}_j^A = \mathrm{A}_j$ *AND* B_j, $j=1,\dots,m$ are composite estimates predicates.

The following lemma defines the conditions under which we can obtain the maximum degree of deducibility by using implications [10, 11, 12].

Lemma. Let $A_1(x),\dots,A_m(x)$ and $B_1(y),\dots,B_m(y)$ where x is the variable of sort l_1, y is the variable of sort l_2 are formulas. Let

$$T=\{a_{j,t}/A_{j,x}[t],\, c_{j,t}/A_{j,x}[t]\Rightarrow B_{j,y}[s] \mid j=1,\dots,m,\, t\in M_{l_1}, s\in M_{l_2}\}$$

is the fuzzy theory. Then T is consistent and

$$T \vdash B_{k,y}[s],\ \ b_{k,s}=\bigvee_{t\in M_{l_1}}\left(a_{j,t}\otimes c_{j,t}\right),\quad s\in M_{l_2}. \qquad (2)$$

Proof. There exists a model D [12] that

$\mathrm{D}\left(A_{j,x}[t]\right)=a_{j,t}$, $\mathrm{D}\left(B_{j,y}[s]\right)=b_{j,s}$, $j=1,\dots,m$.

We must show that D $\models$T.

Obviously, the

$a_{j,t} \otimes c_{j,s} \leq \vee_{t \in M_{l1}} \left(a_{j,t} \otimes c_{j,s} \right)$

is true for all $t \in M_{l1}, s \in M_{l2,}\ j = 1,...,m$. In addition, we find that

$c_{j,s} \leq a_{j,t} \to \vee_{t \in M_{l1}} \left(a_{j,t} \otimes c_{j,s} \right) = \mathrm{D}\left(A_{j,x}[t] \right) \to \mathrm{D}\left(B_{j,y}[s] \right) = \mathrm{D}\left(A_{j,x}[t] \right) \Rightarrow \left(B_{j,y}[s] \right)$

for all $j = 1,...,m$, and, consequently, D $\models$ *T* is a model of theory *T*, which implies that *T* is consistent.

Consider the deductions

$c_{k,s} / A_{j,x[s]}[t] \Rightarrow B_{j,y}[s]$ - the special axiom,

$a_{k,t} \otimes c_{k,s} / B_{k,y}[s]$- modus ponens

for all $t \in M_{l1}, s \in M_{l2,}\ j = 1,...,m$. Then

$b_{k,s} \geq \vee_{t \in M_{l1}} \left(a_{j,t} \otimes c_{j,s} \right).$

But at the same time

$b_{k,s} \leq \vee_{t \in M_{l1}} \left(a_{j,t} \otimes c_{j,s} \right),$

which implies (2) because of the completeness theorem.

This lemma determines the degree of deducibility in the case when some relation is obtained by disjunction of conjunctions of certain formulas.

From these considerations it follows that there are two methods in FLb with linguistic variable. The first of these works with a linguistic description LD^I, composed of logical implication formulated at the natural language, and is used when necessary to withdraw approval of certain facts. The second method is based on the additional assumption and works with a linguistic description LD^A, which consists of the conjunctions of linguistic predicates and is used when there is a need to describe the relation or function. In this paper description LD^A is not considered.

The following theorem proves that, when considering IF-THEN rules as conditional clauses, which are formed from simple estimated predicates, the rule of deduction based on them gives the best deduction in the fuzzy theory, defined by them [12,13, 14].

Theorem. Let we have a linguistic description LD^I, consisting of *m*- IF-THEN rules and the estimated statements A_k from any rule of LD^I. Let A'_k is the possible modification of A_k such that the direction of its intensity $\mathrm{A}'_{k,<x>}$ may vary within the limits of $\mathrm{A}_{k,<x>}$ in some estimates. Let

$\mathrm{T} = \{\mathrm{A}'_k, \mathrm{LD}^I\}$

is a theory in FLb. Then we can get a deduction with intensity of B'_k

$$\mathrm{B}'_{k,<y>} = \left\{ b'_{k,s} = \bigvee_{t \in M_{l_1}} \left(a'_{k,t} \otimes c_{k,s} \right) / B_{k,y}[s] \mid s \in M_{l_2} \right\},$$

such that all $b'_{k,s}$ in multiformula $\mathrm{B}'_{k,<y>}$ are maximal.

Proof. The theory T determines fuzzy theory

$$T = \{A'_{k,<x>}, A_{j<x>} \Rightarrow B_{j,<y>} \mid j = 1,...m\}$$
$$\{a_{k,t} / A_{k,x}[t], c_{j,s} / A_{j,x}[t] \Rightarrow B_{j,y}[s] \mid k \in [1,m], t \in M_{l_1}, s \in M_{l_2}\},$$

where $c_{j,s} = a_{j,t} \to b_{j,s}$. Since IF-THEN rules from LD^I consist of a simple estimated predicates, they can be translated into terms of the language J. Then the statement of this theorem follows from the above lemma.

4. Conclusion.

In this paper we consider within FLb the problem modeling of informative combination of the object's features, receiving language values. Based on the introduced concepts objects are described by syntagms, the interpretation of informative combination. The way for eliminating the existing gap between the application of natural language, resources of its interpretation and support of an accurate expression of the meaning of statements because of understanding of natural language was more meaningful.

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Mamdani biliklər bazasında qeyri-səlis məntiq qaydaları ilə yaxınlaşmalar

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XÜLASƏ

Mamdani yaxınlaşmaları xüsusi aksiomlardan istifadə etməklə üçün qeyri-səlis qaydaların alinması məsələsinə baxılır. Baxılan məsələnin həlli üçün qeyri-səlis məntiqdən geniş istifadə olunur. Dil sintaqmalarının xüsusi siniflərini təyin edən tanıma usulu təbii dilə əsaslanır. Tanımanı həyata keçirən informativ əlamətlərin kombinasiyası metoduna əsaslanan model təklif olunmuşdur.

Açar sözlər: geniş mənada qeyri-səlis məntiq, sintaqma, predikat, intensivlik, linqivistik təsvir.

Приближение на основе базы знаний Мамдани и нечетких правил

У.Ш. Рзаева

РЕЗЮМЕ

Рассматривается задача получения нечетких правил аппроксимации Мамдани в качестве вычета специальных аксиом. Для решения этой проблемы широко используется нечеткая логика. Метод распознавания, с помощью которого определяется особый класс языковой синтагмы, основан на естественном языке. Также представлена модель методом комбинации информативных признаков, с помощью которых происходит распознавание.

Ключевые слова: нечеткая логика в широком смысле, синтагмы, предикат, интенсивность, лингвистическое описание.